ACCELERATION AND SOME LOCOMOTIVE PROBLEMS.

By William G. Raymond.

There are certain locomotive problems of interest to the railroad locating and maintenance engineer, which involve the tractive power of the locomotive, train resistance, and the general principles of accelerated motion.

The most important of these problems are:

1. To determine the load a given locomotive can haul on a given grade at a given constant velocity.
2. To determine the distance on a given grade required by a given locomotive with a given load, to increase or diminish its speed from one given velocity to another.
3. The time required to produce this change in velocity.
4. The converse of 2, to determine what velocity can be acquired by a given locomotive with a given load in a given distance, on a given grade.
5. To determine the length of grade steeper than that for which a locomotive is loaded that can nevertheless be ascended by the aid of a run at the hill. This is the problem of velocity or momentum grades.
6. To determine the speed of a given locomotive with a given load throughout a given division of track.
7. To specify the principal features of the locomotive to be purchased for a given service.

In all these problems certain data which in the nature of the case are to some extent indeterminate must be assumed to be exactly known, and certain other conditions must be assumed that in reality never exist, with the result that practically all of the determinations are approximate and must be considered to be of the nature of carefully prepared estimates. This is particularly true in developing formulas applicable to different locomotives for which the several constants have not been determined by road tests.

ACCELERATION.

Accelerated motion plays an important part in practically all of these problems, and there follows a brief statement of principles and the development of working formulas.

By the property of inertia, all bodies tend to stay in that condition of motion in which at any instant they may be. An accelerating, retarding, or deviating force must be applied to change the condition of motion as to velocity or direction.

It is known that a constantly applied force of given magnitude will produce a uniformly changing condition of motion. The rate of change is called the acceleration and may be positive or negative (retardation). It is known also that the acceleration of a given mass is proportional to the magnitude of the constant unbalanced force acting. Thus, if w be the weight of a body, i. e., the measure of the force of gravity acting on it, and g be the acceleration due to gravity, and if P be any other force applied to the body, the acceleration a, produced by P, will be given by

$$a = \frac{P}{w} g \qquad (1)$$

from which the force P necessary to produce the acceleration a in a body of weight w is

$$P = \frac{w\,a}{g} \qquad (2)$$

Under the influence of the force of gravity the velocity of a falling body increases g feet per second, g having a value varying with the distance from the center of mass of the earth and with latitude, but usually assumed for mechanical problems as 32.16. If the body start from rest it will have a velocity of g feet at the end of the first second, its average velocity for the first second will therefore be $\frac{g}{2}$ feet which will also be the space covered in the first second. At the end of t seconds the velocity will be t g feet per second, the average velocity will have been $\frac{t\,g}{2}$, and the space passed over will therefore be $\frac{t\,g}{2} \times t = \frac{t^2 g}{2}$ feet. If v be velocity in feet per second, t be time in seconds, and h the space or height of fall,

$$v = g\,t \qquad (3)$$

$$h = \frac{g\,t^2}{2} \qquad (4)$$

Since from (3) $t = \frac{v}{g}$, substitution in (4) gives

$$h = \frac{v^2}{2\,g} \qquad (5)$$

Perfectly analogous to these equations, if P be a force acting on a body and producing an acceleration of a feet per second, for t seconds, covering a space of l feet

$$v = a\,t \qquad (6)$$

$$l = \frac{a\,t^2}{2} \qquad (7)$$

$$l = \frac{v^2}{2\,a} \qquad (8)$$

If a body be uniformly accelerated in a distance of l feet from rest to a velocity of v feet per second, the acceleration from (8) is

$$a = \frac{v^2}{2\,l} \qquad (9)$$

and the force P necessary to produce this acceleration, given by substituting for a in (2), its value from (9), is

$$P = \frac{w\,v^2}{2\,g\,l} \qquad (10)$$

If the velocity is expressed in miles per hour V

$$v = \frac{5280\ V}{3600}$$

and $$P = \frac{w}{2\,g\,l} \times \left(\frac{5280\ V}{3600}\right)^2$$

and if the weight is expressed in tons W of 2000 pounds

$$w = 2000\ W$$

and $$P = \frac{66.9\ W\ V^2}{l} \qquad (11)$$

If a train be the body, P is the tractive effort to be exerted by the locomotive to produce the velocity of V miles per hour in the distance of l feet.

But not only is the train given a velocity of translation, the wheels are given a velocity of rotation, requiring P to be larger than indicated by the foregoing expression by an amount depending on the relative masses of car and wheels, the pattern of the wheels and the velocities. For any given set of conditions the addition to P may be determined by comparing the energy required to accelerate the car wheels in their motion of rotation with that required to give the resulting motion of translation to the car as a whole. No great precision can be attempted for a general formula. The increase of P may be as little as 2½ per cent, and it may be as high as 6 or 8 per cent over that given by equation (11). Adopting 4.63 per cent for simplicity of result

$$P = 70 \frac{V^2}{L} W \qquad (12)$$

This force P must be in excess of the forces necessary to overcome all other resistances. It is probable that no train is uniformly accelerated from rest to any given velocity it may attain, because from a velocity of 0+ to 5 or 6 miles an hour the pull an engine exerts is nearly constant and is the tractive effort of adhesion,* while the resistances to motion rapidly decrease, leaving an increasing portion of the tractive effort for acceleration. When the velocity of 5 or 6 miles is exceeded the resistances to motion slowly increase, the tractive effort decreases, and there results a decreasing force available for acceleration, decreasing somewhat more rapidly than in proportion to the increase of velocity.

If the velocity is to be increased from V_1 miles per hour to V_2 miles per hour, the force required is

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*This statement may be questioned. The Pennsylvania Railroad testing department, in its estimates of maximum tractive effort of simple locomotives, counts on only 80 per cent of the boiler pressure as available in the cylinders even at minimum speeds. If this allowance is correct, probably no simple locomotives in common use can ever exert their full tractive effort of adhesion, which is usually estimated to be as high as one-fourth of the weight on the drivers for favorable conditions of track, and not usually lower than one-fifth under quite unfavorable conditions.

The Baldwin Locomotive Works states that the initial pressure in the cylinder may vary from full boiler pressure at very slow speeds to 85 per cent of boiler pressure at high speeds of 300 revolutions per minute, and uses rather better than 90 per cent of the initial pressure when the speed is less than 50 revolutions per minute, indicating that the full tractive effort of adhesion may be realized by the cylinders at very slow speed.

Further road tests are perhaps necessary to establish the facts. The locomotive testing plants thus far devised are not adapted to tests of high tractive effort at slow speed.

$$P = 70 \frac{W}{l} (V_2^2 - V_1^2) \qquad \text{(A)}$$

If the force be known, and it is desired to determine the distance required to increase the velocity from V_1 to V_2 miles per hour,

$$l = 70 \frac{W}{P} (V_2^2 - V_1^2) \qquad \text{(B)}$$

If the distance and available force are known, and it is desired to know how great a load can be carried with the required acceleration, solve A or B for W and get

$$W = \frac{P\,l}{70\,(V_2^2 - V_1^2)} \qquad \text{(C)}$$

If W, P, l and V_1 are known, and V_2 is desired, solve for V_2 and get

$$V_2 = \pm \sqrt{\frac{P\,l}{70\,W} + V_1^2} \qquad \text{(D)}$$

In determining l, since P can never be constant, nor even approximately constant, through any considerable change in speed, it is not uncommon to find l for a change in speed of 1 mile per hour, using successively V_1, V_1+1, V_1+2, etc., as initial speeds, until the required change is reached, when the sum of the several values of l will be the distance required. If $V_2 = V_1 + 1$, equation (B) becomes

$$l = 70 \frac{W}{P} (2\,V_1 + 1) \qquad \text{(E)}$$

The load W in any problem likely to arise would be known or assigned from some estimate made as hereinafter indicated. The tractive effort P must be estimated by subtracting from the estimated total tractive effort of the locomotive, the resistance due to such grade as the train may be on, and the ordinary train resistance, an unknown, and in nature an indeterminate quantity.

Many attempts have been made to determine a rational expression or formula for train resistance, but none has yet been devised, nor is it probable that any simple formula ever will be devised that shall correctly give the quantity known as train resistance. It depends on the condition of the journals and the weight on them, on the condition of the rail surface and the weight on it, on the pattern of the wheels, on the condition of the roadbed, on the temperature, on the velocity and direction of the wind, the speed of the train, etc. It is usually estimated at so many pounds per ton of train, and some of the estimates will be given. Grade resistance, being the action of gravity, on an incline may be determined with precision, and is always proportional to the weight of train.

TRACTIVE EFFORT

The tractive effort of the locomotive has three limits; it can not possibly be greater than the tractive effort of adhesion which is the weight on driving wheels multiplied by the coefficient of static friction between wheels and rails; nor can it be greater than the cylinder tractive effort which varies with the steam pressure in the cylinders, the diameter of the pistons, the stroke, and the diameter of the driving wheels. If the cylinders are large enough, the drivers small enough, and the steam pressure high enough, the cylinder tractive effort would equal the tractive effort of adhesion, and the locomotive should be so designed that this is the case at low speeds. As the speed increases the effective pressure in the cylinders falls, and the full tractive effort of adhesion can not be had; moreover, with such boilers as have as yet been devised, the supply of steam at high pressure necessary to give the full tractive effort of adhesion can not be maintained at high speed.

A boiler is capable of developing a more or less definite horse power, and if the work performed is performed at high speed, the force exerted must be relatively small if the power exerted remains constant. Thus we have the boiler tractive effort as the limiting quantity at anything over the lower speeds of from six to ten miles an hour, the precise limit depending on the design of the locomotive.

The coefficient of static friction between wheel and rail is usually estimated at about one-fourth for favorable conditions, as high as one-third with a sanded dry rail, and as low as one-fifth or less for ordinary winter conditions.

In determining P, therefore, for low speeds under six miles an hour, either the tractive effort of adhesion, i. e.,

$$T_a = \text{weight on drivers} \times \text{coefficient of friction}$$

or the cylinder tractive effort should be used, and for higher speeds, either the cylinder tractive effort or the boiler tractive effort. It is practically always true that boiler tractive effort must be used at speeds of over 8 to 10 miles an hour. In any event, the tractive effort that is smallest must be used.

Cylinder tractive effort is given by the formula

$$T_c = \frac{C^2\ L\ p}{D} \qquad \text{(F)}$$

in which C is the diameter of the piston in inches, L the stroke in inches, p the mean effective pressure in the cylinder in pounds, D the diameter of the drivers in inches, and T_c the tractive effort in pounds.

The boiler tractive effort is given by

$$T_b = \frac{375\ I\ H\ P}{S} - F \qquad \text{(G)}$$

in which I H P is indicated horse power, S is speed in miles per hour, F is the machine friction reduced to pounds of tractive effort, and T_b is the tractive effort in pounds. The first term of this formula, although not new, is not so generally known but that its derivation may be given.

One horse power is 33,000 foot-pounds of work per minute, or 60 × 33000 foot-pounds per hour. Assuming no losses from friction the tractive effort multiplied by the distance through which it acts in a given time equals the work done in that time, and this divided by the foot-pounds of work corresponding to one horse-power for the given time, should be the horse power developed by the locomotive. Therefore

$$\frac{T_b \times S \times 5280}{60 \times 33000} = I\ H\ P \text{ or}$$

$$T_b = \frac{375\ I\ H\ P}{S}$$

And since there are losses due to friction of the machine parts that friction must be deducted, giving equation (G).

The horse power of a locomotive is not usually given, but if not known it may be estimated from the heating surface.

The maximum power per square foot of heating surface varies with the design of the locomotive, but recent tests* seem to indicate that simple freight locomotives developing full power produce one cylinder horse power for each 2.3 square feet of heating surface, varying somewhat either side of this average, and that compound locomotives may produce one cylinder horse power for

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* Pennsylvania Railroad tests at the Louisiana Purchase Exposition, which seem to confirm earlier results of Professor Goss.

each 2 feet of heating surface with very decided variation either side of this mean value. Thus, for a simple freight locomotive, the boiler tractive effort may be expressed by

$$T_b = \frac{375 \times \frac{H}{2 \cdot 3}}{S} - F = \frac{163\,H}{S} - F$$

in which H is square feet of heating surface and F includes the rolling resistance of the drivers. F varies with the speed between limits of 6 per cent and 25 per cent of the indicated power, but with the locomotive developing full power, or a little less, it may fairly be taken at 10 to 12 per cent for estimates. In round numbers, therefore, the boiler tractive effort is

$$T_b = \frac{145\ H}{S} \qquad \text{(H)}$$

GRADE RESISTANCE.

Grade resistance, in pounds per ton of train, is given by

$$R_g = 20\ r$$

in which r is the rate of rise in 100 feet, or the rate per cent of the grade. It is a resistance or retarding force to an ascending train and an accelerating force to a descending train.

TRAIN RESISTANCE

The work done in drawing the train behind the tender on a straight, level track, is the product of the quantity called train resistance and the distance through which the train is drawn. Train resistance is usually considered to include all resistance taxing the tractive effort of the drivers, except grade and curve resistances. It includes the rolling resistance of the driving wheels, which in later estimates, because of the character of locomotive testing plants, is included with the machine friction. Train resistance arises from (1) journal friction; (2) rolling friction or resistance; (3) resistance due to oscillation and concussion; (4) head, tail and side resistance of the atmosphere.

Journal friction is a maximum of 15 or 20 pounds per net ton at a velocity of 0+ just after starting from rest; it is not nearly so much when slowing down from motion to 0+ or after a momentary stop. From this maximum it falls rapidly as the velocity increases to an unknown minimum possibly approximating 2 pounds per ton. It is very much affected by temperature, and if a minimum of 2 pounds is realized in summer temperature, it is very probable that the minimum may be 4 to 6 pounds in winter weather. It varies very little with velocity if the speed is above 6 or 8 miles an hour. It depends very much on the character of the lubrication and the condition of the bearings.

Rolling resistance is unknown in amount and is usually classed with journal friction. It doubtless varies much with the condidition of the track, and with the insistent weight, and is little affected by velocity changes. Rolling resistance and journal friction together are assumed at from 2 to 3 pounds per net ton in modern expressions for train resistance.

Resistance due to oscillation and concussion is unknown in amount, is believed to be very small, and probably varies with the square of the velocity.

Atmospheric resistance has been most thoroughly investigated by Professor Goss at the Purdue laboratory. Much depends on the form of the cars and the make-up of the train. A freight train of box cars moving through still air seems to be resisted by a force given by the expression $A = (13 + .01\ C)\ V^2$, C being the number of cars in the train. For the engine and tender alone $A = .11\ V^2$ and for the train alone $A = (.016 + .01\ C)\ V^2$. For passenger trains the coefficient of C is to be doubled. At ordinary freight train speeds the whole quantity is small, but at high velocities the resistance is considerable, consuming from ten to twenty per cent of the tractive force of the locomotive. The foregoing values are for motion through still air. A head wind of velocity equal to that of the train would increase the resistance four times, a side wind would have an unknown effect which would be quite large.

Summarizing all we know of train resistance, it is probable that the whole may be represented by an equation of the form

$$R = \left(A + B\,V + \frac{C}{(V + K)^2} + D\,V^2\right)W + M\,V^2$$

in which R is the total resistance in pounds, A, B, C, D, K and M are coefficients, some of which may be zero, to be determined by experiment, W the weight of the train, and V is the velocity in miles per hour. W being expressed in tons, the parenthesis gives the resistance in pounds per ton of train, which is the usual way of stating it.

The commoner formulas for train resistance are much simpler than that just given.

The Baldwin Locomotive Works formula is

$$R_T = 3 + \frac{V}{6}$$

in which R_T is resistance in pounds per ton of train and V is speed in miles per hour.

The Engineering News formula is

$$R_T = 2 + \frac{V}{4}$$

These formulas make no allowance for the fact that loaded trains have a less resistance per ton than empty trains, and they also probably include the machine friction of the locomotive. In using these formulas in connection with the boiler tractive effort, the undiminished value of the tractive effort should be used, namely $T_b = \frac{163\ H}{S}$ or $\frac{375\ I\,H\,P}{S}$

Formulas that are perhaps better for freight train resistance are those devised by Mr. Sanford L. Cluett to fit the curves of the late Mr. A. M. Wellington; they are

For empty trains $R_T = 5.4 + 0.01\ S^2 + \frac{70}{(S+3)^2}$

For loaded trains $R_T = 3.8 + 0.0076\ S^2 + \frac{16.4}{(S+1)^2}$

The formulas give results probably much too great for high speeds, and possibly somewhat too high for all speeds. The following modifications are suggested, and while less simple than the Engineering News or Baldwin formulas, they are believed to fairly well fit freight train resistance curves, not including machine friction, and are applicable for speeds of from 0+ to about 35 miles an hour.

Loaded train $R_T = 3.5 + 0.0055\ S^2 + \frac{16}{(S+1)^2}$

Empty train $R_T = 5.0 + 0.007\ S^2 + \frac{8}{(S+1)^2}$

CURVE RESISTANCE

Curve resistance is usually estimated at about $\frac{1}{3}$ of a pound per ton of train per degree of curve. That is, a 4 degree curve will offer a resistance of 4/3 pounds for each ton of train on the curve.

SOLUTION OF PROBLEMS.

Having stated the fundamental formulas, it remains to indicate their use in solving the problems mentioned in the beginning of this paper.

In advance, one formula for train resistance is adopted and diagrammed or tabulated so that the resistance for any speed may be taken at once from the diagram or table. It will perhaps be best if several diagrams or tables are made for various percentages of loading on the train, the several curves for partial loading lying between the curves of loaded and empty trains.

Next, for the particular locomotive to be discussed, a diagram or table of tractive effort should be made, using the boiler tractive effort formula for all speeds above that for which the boiler tractive effort equals the cylinder tractive effort, using, say, 80 per cent of boiler pressure as mean effective pressure in the cylinder formula, except that for speeds below 5 miles an hour 85 per cent of boiler pressure may probably be safely used. Should the tractive effort of adhesion be less than the cylinder tractive effort that quantity should be used for the lower speeds.

Problem 1. To find the load a given locomotive can haul on a given grade at a given constant velocity. The sum of the resistances must equal the tractive effort; therefore, the tractive effort at the assumed speed should be placed equal to the train resistance and grade resistance, indicated by the weight times the resistance in pounds per ton, and the weight obtained thus:

$$T = W\,(R_T + 20\,r)$$

$$W = \frac{T}{R_T + 20\,r}$$

If r is the ruling grade of the road, T should be taken for the minimum allowable speed; which is placed by different experimenters at from 5 miles per hour to 10 miles per hour. It should be that speed which is just enough to make stalling from small accidents of firing, or track condition, unlikely. When this speed is used the rating for the locomotive over the division is obtained.

Problem 2. Assuming this load, let it be required to determine the distance on some grade less than the ruling grade in which, if the locomotive exerts its full power, the velocity may be increased by one mile per hour. Again, the net tractive effort must equal the sum of the resistances including that due to acceleration. The P of equation (E) becomes T — W (R_T + 20 r). T and R_T should be values either for S_1 or ($S_1 + \frac{1}{2}$) or (S_1 + 1). It will be more nearly exact to consider T and R_T the tractive effort and resistance for ($S_1 + \frac{1}{2}$). Then (E) becomes

$$1 = 70 \frac{W}{T - W(R_T + 20\,r)} (2\,S_1 + 1)$$

$$1 = \frac{70\,(2\,S_1 + 1)}{\frac{T}{W} - (R_T + 20\,r)} \qquad \text{(I)}$$

If it is desired to find the space required to increase the velocity from S_1 to S_2 miles per hour, equation (I) is solved with successive values of S and corresponding values of T and R_T until the value of S_2 is reached; the values of l thus found are added for the required result.

If it is desired to know what average speed may be made up such a grade of length L, equation (I) is solved with successive values of S and corresponding values of T and R until the sum of the several values of l equals the length L; then for an approximate result average the initial and final speeds. For a more exact result each S or each ($S + \frac{1}{2}$) is multiplied by the corresponding l, the products summed, and the sum divided by L. The sum of the l's will probably not just equal L, but extreme precision is useless in such a problem, since the assumed conditions are rarely those obtaining; the whole train does not enter the grade at once, may never be on the grade, and does not leave it at once.

Both of these problems may be approximately solved by using for T and R_T their values for the mean velocity, known in the first and estimated in the second, and substituting the resulting P in equations (B) and (D).

Problem 4. The procedure is as in the last problem, omitting the averaging.

Problem 5. A locomotive and train approaches a grade steeper than the ruling grade for which it is loaded at a speed of S_1 miles an hour, and may leave it at a speed of S_2 (less than S_1) miles an hour. How long may the grade be?

P of equation (B) is now essentially a negative or retarding force, and the parenthesis ($V_2^2 - V_1^2$) becomes for $V_2 = S_2 - 1$, — ($2\,S_1 - 1$) therefore (E) becomes

$$1 = 70 \frac{W}{P} (2\,S_1 - 1)$$

and as before P is the difference between T for speed, S_1 or $S_1 - \frac{1}{2}$, R_T for the same speed and R_g, or

$$l = 70 \frac{W}{T - W\,(R_T + 20\,r)}\,(2\,S_1 - 1)$$

$$= \frac{70\,(2\,S_1 - 1)}{\frac{T}{W} - (R_T + 20\,r)} \qquad (K)$$

Solving with successive values of S to S_2 and corresponding values for T and R_T, and summing the results, the possible length of grade is obtained. Again, an approximate solution may be had by substitution in (B), using for P a value obtained by taking T and R_T at their values for the mean velocity on the grade, thus:

$$l = \frac{70\,W\,(S_2^2 - S_1^2)}{T - W\,(R_T + 20\,r)}$$

$$= \frac{70\,(S_2^2 - S_1^2)}{\frac{T}{W} - (R_T + 20\,r)} \qquad (L)$$

If both numerator and denominator of (K) and (L) be divided by 20, there results

$$l = \frac{3.5\,(2\,S_1 - 1)}{\frac{1}{20}\left(\frac{T}{W} - R_T\right) - r} \qquad (K')$$

$$l = \frac{3.5\,(S_2^2 - S_1^2)}{\frac{1}{20}\left(\frac{T}{W} - R_T\right) - r} \qquad (L')$$

But from the equations given under Problem 1

$$T = W\,(R_T + 20\,r) \quad \text{and}$$

$$r = \frac{1}{20}\left(\frac{T}{W} - R_T\right)$$

is the grade on which with tractive effort T and train resistance R_T, the locomotive can just draw the train at the constant velocity corresponding to T and R_T, and this quantity is seen to be the first term of the denominator in both (K′) and (L′). If, therefore, r′ be the grade on which the locomotive can just draw the weight W at the mean velocity of $(S_1 + \frac{1}{2})$ or $\frac{S_1 + S_2}{2}$, then (K′) and (L′) become

$$l = \frac{3.5\,(2\,S_1 - 1)}{r' - r} \qquad (M)$$

$$= \frac{3.5\,(S^2_2 - S_1^2)}{r' - r} \qquad (N)$$

The quantity r′ may be tabulated in advance for the given locomotive when the problem of determining l for any grade on the road will be a very simple matter. r′ is known as the virtual grade.

Equation (I) may be similarly treated and will then become

$$l = \frac{3.5(2\,S_1 + 1)}{r' - r} \qquad (P)$$

Problem 6. This problem is determined by successive solutions of the preceding problems. Assuming the locomotive loaded for minimum speed on the ruling grade of the division, the average speed that can be made on the several grades is determined taking care that a maximum speed of, say, 30 miles an hour, is never exceeded, and introducing all probable stops, The average speed over the division is then readily determined. This will be in the nature of things an estimate and should be checked by trial on the road.

Problem 7. Problem 7 is quite complex. The method of investigation may be stated somewhat as follows. Determine the load to be hauled on the ruling grade at the minimum speed; from the maximum tractive effort required determine the weight on drivers, and from the allowable unit weight the number of drivers (note that the result is rational and practicable); determine a desirable average schedule time, and with this and some grade on the division assumed at average speed to require the full capacity of the locomotive, and the determined load, estimate the total resistance and the necessary power to overcome this at the assumed speed; with this assumed power, a profile of the division, and the determined load, find the speeds at which the various grades can be worked, compare the resulting average with that deemed desirable and modify the power as may be necessary; determine the heating surface, and approximate dimensions of boiler by comparison with existing locomotives or *de novo*; see that the results are practicable; state the requirements to be a locomotive with the determined weight on drivers and the determined cylinder horse power, to be developed most economically at the determined average speed, to be capable of such a maximum speed, and with such a maximum cylinder tractive effort as has been determined, and leave the proportioning to the locomotive designer, checking the design in these particulars when it shall be submitted.

Problem 3. The time required to gain the velocity v feet per second from rest, if gained in the distance l feet is

$$t = l \div \frac{v}{2}$$

with v expressed in miles per hour S, since

$$v = \frac{5280}{3600} S$$

$$t = \frac{15}{11} \frac{l}{S}$$

and if the speed is to be increased from S to S_2 miles per hour in the distance l feet, the time required is

$$\text{t seconds} = \frac{15}{11} \frac{l}{S_1 + S_2} \qquad \text{(R)}$$

Hence, having found the distance required for a given acceleration or retardation, substitute it for l in equation (R) and solve for the time.

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