# SEMI-ANALYTIC ESTIMATION OF THE RESPONSE OF HAND-HELD TOOLS AND ITS APPLICATIONS

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## Introduction

Many hand-arm models have been proposed by researchers for various purposes, which include lumped parameter or continuous models[1]. In this paper it is shown that a semi-analytic response solution of a hand-held tool can be obtained by using a given hand-arm model and the tool vibration force estimated from measurement. This semi-analytic solution can be compared with the measured response of the hand-held tool. The approach can be used to compare performance of hand-arm models, and also to estimate the force transmitted to the hand.

## Methods

Fig. 1 (a) shows a tool running free and suspended by a very soft bungee cord. Because the spring constant $k$ is small, the response of the tool is:

$$\boldsymbol{X}(\omega) = \frac{\boldsymbol{F}(\omega)}{k-\omega^2 M_{tool}} \cong -\frac{\boldsymbol{F}(\omega)}{\omega^2 M_{tool}} = -\frac{\boldsymbol{A}(\omega)}{\omega^2} \quad (1)$$

From Eq. (1), $\boldsymbol{F}(\omega) = M_{tool}\boldsymbol{A}(\omega)$; therefore the tool vibration force can be estimated from the measured acceleration. It is noted that the tool vibration force is largely a result of unbalanced mass as illustrated in Fig. 1 (a).

Fig. 1 (b) describes the motion of a free-running tool held by a hand-arm modeled as a three D.O.F system. The equation of motion is described as follows;

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \quad (2)$$

where, the mass corresponding to $x_1$ should be taken as $M_1$+$M_{tool}$. In the frequency domain, Eq. (2) becomes

$$[K - \omega^2 M + j\omega C]\{\boldsymbol{X}(\omega)\} = \{\boldsymbol{F}(\omega)\} \quad (3)$$

As mentioned above, $\{\boldsymbol{F}(\omega)\} = [M_{tool}\boldsymbol{A}(\omega), 0,0]^T$ is obtained from measurement. Therefore,

$$\{\boldsymbol{X}(\omega)\} = [K - \omega^2 M + j\omega C]^{-1}\{\boldsymbol{F}(\omega)\} = [\boldsymbol{H}(\omega)]\{\boldsymbol{F}(\omega)\} \quad (4)$$

$\{\boldsymbol{X}(\omega)\}$ obtained as such can be considered as a semi-analytic solution because it is the response calculated from the theoretical model using the force estimated experimentally.

## Discussions

This semi-analytically estimated response obtained by the above-explained procedure can be used to evaluate performance of a given hand-arm model. Fig.2(a) compares semi-analytically estimated accelerations in the x-direction obtained from 5 different lumped parameter models with the directly measured acceleration of an angle grinder operating hand-held. Figure 2(b) compares measured and simulated acceleration time histories of the tool. The approach can also be used to estimate the tool force transmitted to the hand-arm system. As illustrated in Fig. 1 (c), the force can be calculated by:

$$\boldsymbol{F}_{hand}(\omega) = \boldsymbol{Z}_1(\omega)\frac{\boldsymbol{A}(\omega)}{j\omega} \quad (5)$$

where, $\boldsymbol{Z}_1(\omega)$ is the impedance calculated from the hand-arm model and $\boldsymbol{A}(\omega)$ is the measured acceleration of the tool.

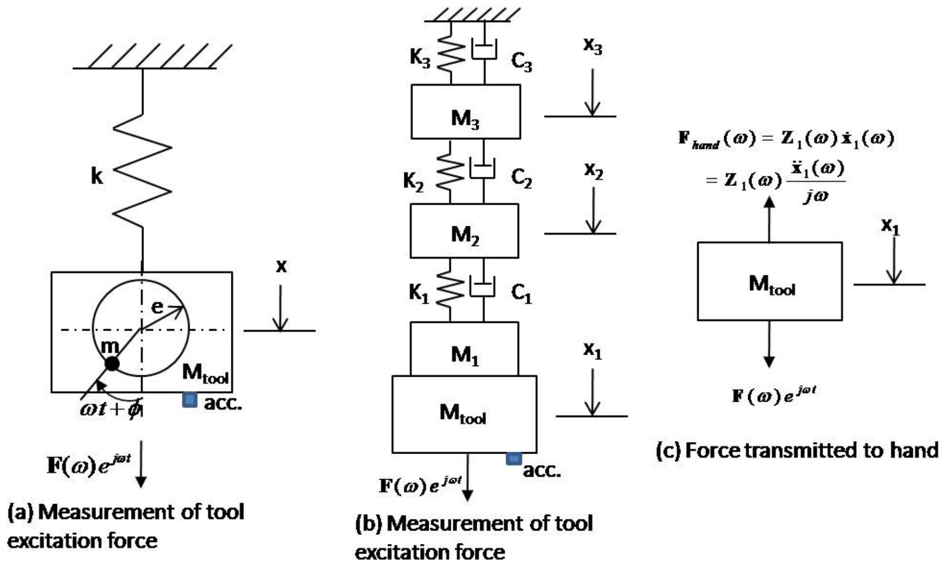
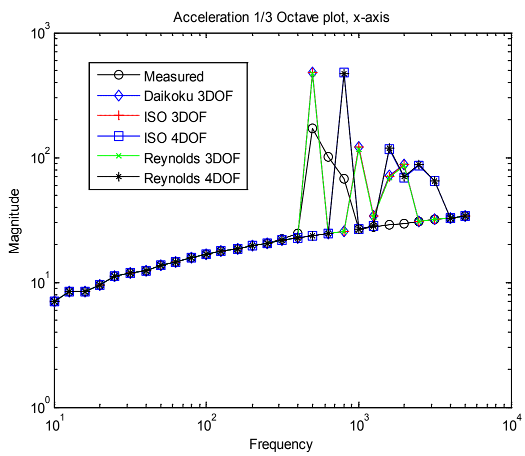
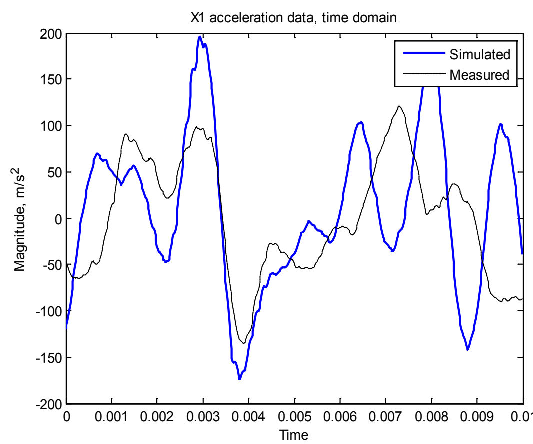



Fig.1 Concept of semi-analytic estimation of the operating response of the tool and hand-transmitted force




(a) Comparison of accelerations calculated from 5 models with measurement in 1/3 octave spectra




(b) Measured and simulated acceleration time histories from the 3 DOF model proposed by Reynolds

Fig. 2 Comparison of measured and semi-analytically estimated x-direction accelerations of an angle grinder.

## References


1. N. Harada and M.H. Mahbub 2007 *Presented at the 2nd International Workshop on diagnosis of hand-arm vibration syndrome. Göteborg, Sweden.* Diagnosis of vascular injuries caused by hand-transmitted vibration.

2.S. Rakheja, J. Z. Wu, R. G. Dong and A. W. Schopper 2002 in *Journal of Sound and Vibration*. A comparison of biodynamic models of the human hand-arm system for applications to hand-held power tools.